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Discussion of
"THE BOX INLET DROP SPILLWAY AND ITS OUTLET"

by Fred W. Blaisdell and Charles A. Donnelly
(Proc. Paper 534)

FRED W. BLAISDELL,[1] M. ASCE, AND CHARLES A. DONNELLY.[2]—The writers greatly appreciate Mr. Minshall's discussion. He neglected to mention that his association with this type of structure dates from 1933, when he worked with L. H. Kessler in making some early tests.[3] Mr. Minshall also worked on the design and construction of many of the box inlet drop spillways built in western Wisconsin in the 1930's. He has already mentioned the evaluation of their performance which he conducted in 1949,[11] Mr. Minshall is, therefore, well qualified to discuss the writers' paper.

The studies reported by the writers were made to obtain design information for use by the Soil Conservation Service, and the limits of the test variables, including the limitation of D/W to a maximum of 1.0 were largely set by the SCS design engineers. It is unfortunate, as Mr. Minshall points out, that information was not obtained that could be used for estimating the peak discharges through the older structures having higher values of D/W.

The data on the effect of the position of the dike, on which Table 4 is based, were scanty, as the writers were careful to emphasize. Time limitations and a need for concentration of efforts on other studies made it impossible to give the effect of dike position the study it deserved. It appears that the assumptions used to extrapolate Table 4 beyond the range of the tests were not completely correct. For these reasons, the additional data presented by Mr. Minshall are most welcome.

Mr. Minshall states that for B/W = 0 (straight drop spillway) the greatest capacity of the spillway is reached when X/H = 0.5. This statement is at least qualitatively verified by tests made by the junior writer at the St. Anthony Falls Hydraulic Laboratory. These tests, made with vertical parallel approach channel walls, show that the maximum capacity of the straight drop spillway is reached when the width of the approach channel is 1.25 times the length of the straight drop spillway crest.

It is unfortunate that several of Mr. Minshall's comments indicate that he has assumed that Fig. 9 and Eq. 6 apply to the entire curve of Fig. 6. Figure 9 and Eq. 6 apply only to that portion of the curve of Fig. 6 when the control is at the box inlet crest. Equation 3 applies when the control is at the headwall opening. This latter equation includes a correction for the fact that the straight line representing the high head data of Fig. 6 does not pass through the origin. A constant coefficient c_2 is obtained when this H_{02} correction is applied, as is indicated by the straight line in Fig. 6 when the control is at the headwall opening. All of this is pointed out in the first paragraph

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of the section "Control at Headwall Opening." Mr. Minshall's use of Eq. 4 for both crest and drown or submerged crest control undoubtedly accounts for the reduction in c_3 at the higher heads shown in Fig. 25.

The writers' experiments show that the shift from crest control to headwall control takes place when the crest becomes affected by submergence, as Mr. Minshall has shown in Fig. 24, and that at higher heads a constant coefficient is obtained if Eq. 3 is used. The data show that this shift may occur at H/W values ranging from 0.1 to 1.4—the maximum value of H/W included in the test range. This shows that the shift from crest control to headwall control does not occur at a definite value of H/W. The writers' data also show that headwall control sometimes begins at H/W values considerably greater than 0.5, contrary to Mr. Minshall's statement. Values of H/W in excess of 0.5 were obtained for tests 82a to 84, 114 to 122, and 124 to 129, all inclusive. It will be noted from Table 3 that these tests include those conducted on the narrower approach channels and the deepest box inlets.

The writers have observed that the toe of the dike is frequently pushed closer to the crest during construction than is called for on the plans. This reduces the capacity of the spillway below the design value. When a trapezoidal weir box inlet is used, the sloping sidewall portion of the crest provides a definite line to which the fill can be graded. This helps to insure that the construction is carried out according to the design. Mr. Minshall's Eq. 15, which gives the discharge for this type of weir crest when the fill slope is 1 on 3, is, therefore, a welcome contribution.

Some of the thoughts expressed in the preceding paragraph regarding the proximity of the dike toe to the box inlet crest were in the minds of the writers when they made the statement that it is important to keep the toe of the dike well away from the crest. Mr. Minshall has correctly pointed out that the designer must provide the most economical structure. The hydraulic criteria presented in the paper should provide sufficient latitude to permit the determination of which of the several possible combinations of dimensions will fit the site and achieve this end.

The writers are in full agreement that the effect of silting gradients on the discharge capacity should be determined. Other items which might be considered are the effects of channeling through the silt deposits and the effects of lack of maintenance on the capacity.

Mr. Minshall's comments have been both interesting and constructive. The writers thank him for this contribution from his experiences.

Discussion of
"HYDRAULICS OF THE FREE OVERFALL"

by A. Fathy and Mahmoud Shaarawi Amin
(Proc. Paper 564)

A. FATHY[1] AND MAHMOUD SHAARAWI AMIN,[2] A.M. ASCE.—The writers have gone carefully through the discussions of their paper by Mr. Koloseus and by Messrs. Carstens and Carter. They find that in both discussions some really pertinent questions have been raised, but they regret to have to say that the second one contains many statements that need revision. Messrs. Carstens and Carter profess to have come forward with their review to consolidate data quoted from other sources and to point out several errors in the writers' analysis. When certain faulty notions they have utilised are corrected,it will be found that they have done neither.

It would take too long to go over the points raised in the discussions one by one. The best the writers find they can do to clear up the situation is to give a systematic account of the problem from first principles, drawing attention, in passing, to essential facts. Every reader can then draw his own conclusions.

Before doing so, however, there are two special corrections to be made, one in the original paper and the other in the second discussion.

In the original paper Eqs. 11 and 12 have been incorrectly reproduced. They should read:

$$\frac{p_s - p_n}{r_o} - \frac{dp_s}{ds} + \frac{d\tau}{dn} + \frac{2\tau}{r_f} + \gamma \sin\sigma - \rho\frac{v^2}{r_o} = 0 \qquad (11)$$

$$\frac{p_s - p_n}{r_f} - \frac{dp_n}{dn} + \frac{d\tau}{ds} - \frac{2\tau}{r_o} - \gamma \cos\sigma + \rho\frac{v^2}{r_f} = 0 \qquad (12)$$

It is clear that Eq. 13 could not have been deduced from 12 otherwise.

In the second discussion, the values assigned to the critical slope and the Chezy coefficient in the writers' experiments are considerably in error. Presumably, these values were based on the writers' estimate of the coefficient of resistance f for the terminal section in the sample set of figures quoted on page 5 of the original paper. It appears, however, that f was not interpreted according to the writers' notation as given on page 7. Consequently, the points supposed to represent the writers' data in the reviewers' Fig. 5 have been displaced a long way to the right through magnification of the coefficient C. It may be noted, by the way, that the coefficient of resistance is not an absolute constant for the channel as has apparently been presumed in the compilation of the said figure. Actually it varies with both the discharge and the bed slope. Furthermore, it is unlikely to have the same value (even for the same discharge) in the parallel flow zone as at the terminal section. The conception of Fig. 5 is, therefore, wholly erroneous.

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Revision of Theory

Let steady drop-down flow towards a free overfall take place in a long channel as illustrated in Fig. 8. The flow may be generally divided into four zones as follows:

Zone A - wherein the flow is practically uniform.
Zone B - wherein the flow is non-uniform but approximately hydrostatic.
Zone C - wherein the flow is non-uniform and non-hydrostatic.
Zone D - wherein the flow is free.

Now let us consider the flow characteristics in each of these zones in succession.

Zone A - In this zone the kinetic and pressure energies remain unchanged and the position energy lost is totally absorbed by frictional resistance. Putting S_e = slope of energy line and $N = q^2/gy^3$,

$$S_e = S_o = N f/2 \text{ , where } f = \frac{2\tau_o}{\rho V^2} \tag{14}$$

The correction factor β for non-uniform velocity distribution may be supposed incorporated in f. In general, f is a function of y and S_o in addition to boundary roughness and the physical properties of the fluid. The critical slope,

$$S_c = f/2 \tag{15}$$

Zone B - In this zone there is a gain in kinetic energy and a loss in potential (pressure + position) energy. On the supposition of hydrostatic pressure conditions, the potential energy head may be taken as the height (z + y) of the free surface above datum. The energy exchange is subject to two controls: (a) The change in potential energy is linked up with the condition of continuity (q = vy), and (b) The difference between the potential energy lost and the kinetic energy gained in a given distance must equal the energy absorbed by frictional resistance over that distance. The rate of change of effective head is:

$$S_e = S_o - \frac{dY}{dx}(1 - \beta N) - NY\frac{d\beta}{dx} \tag{16}$$

Since the relation between T_o and the mean velocity V must depend on velocity distribution which, in turn, is affected by the rate of convergence (or change of depth generally), the coefficient of resistance must be a function of (dy/dx) in addition to the previously mentioned parameters. In turbulent flow, $\beta = 1$ very nearly and $d\beta/dx$ may be put = 0, so long as dy/dx is small. The equation of motion becomes:

$$S_o - \frac{dY}{dx}(1 - N) - N\frac{f}{2} = 0 \tag{17}$$

whence:

$$\frac{dY}{dx} = \frac{S_o - Nf/2}{1 - N} = \frac{S_o - S_e}{1 - N} \tag{18}$$

which is the ordinary varied-flow equation.

According to this equation the surface slope would be infinite (so long as the flow is non-uniform) at the section where the flow depth equals the critical depth for parallel flow (N = 1). It follows that the critical depth position can never fall in the hydrostatic zone.

At this juncture, it may be opportune to draw attention to the physical implications of the notions of "Critical Depth" and "Specific Energy." The specific energy is commonly defined as the average energy head referred to a datum line passing through the bottom of the channel section, irrespective of energy variations in the direction of flow. Thus it represents the effective energy only in the case of a channel with a horizontal bed. Its significance, however, rests on other considerations than those relating to continuous flow. It will be noted that in impulsive, or discontinuous, changes of depth, the body and the friction forces vanish and it is immaterial whether the channel is horizontal or sloping. The state of minimum specific energy, or of critical flow, is simply the state at which no impulsive change of depth can take place. The specific energy has no place in the analysis of continuously varying flow.

Zone C - In this zone the flow gains one degree of freedom. The rate of change of potential energy is no longer tied up unshakably with the condition of continuity. In other words, changes of potential energy become more or less independent of changes of depth. The friction control, however, remains. But the freedom of the flow to assume new pressure conditions is not absolute. If it were, the flow pattern would be indeterminate. Actually, stability is attained through a combination of inter-related changes in the geometrical, the dynamical and the frictional conditions.

Using the notation of the original paper, the slope of the effective energy line in this case becomes:

$$S_e = N\frac{f}{2} = S_o - \frac{dY}{dx}(\alpha - \beta N) - \frac{Y}{2}\frac{d\alpha}{dx} - NY\frac{d\beta}{dx} \qquad (19)$$

whence Eq. 7.

The parameters α and β play a conjoint part in the self regulation of the flow to accommodate itself to the conditions required at the fall.

As already hinted at, the critical depth position must fall within this zone, if it is passed at all, bearing in mind that the terminal depth is found experimentally to be always appreciably smaller than the critical depth. The writers have used the latter as reference in their analysis not because it was supposed to possess any distinctive physical property, but simply because it constituted a convenient parameter for the expression of the relation between Y_t and q in dimensionless terms.

At the critical depth position, N = 1 and S_e = 0.5f. At the terminal section in a horizontal channel, the observed value of N is 2.77, whence S_e = 1.38f. Apart from the probable increase in the value of f as the fall is approached, this shows that there is a considerable increase in the rate of loss of effective energy between the critical depth position and the terminal section.

A remarkable fact is that where the initial uniform flow depth is subcritical zone B presumably disappears and the flow passes directly from zone A to Zone C. The only alternative is that the entire flow should become non-hydrostatic. This, however, would be difficult to explain in the case of a long approach channel, since appreciable curvature and convergence effects could not extend very far upstream. There is no question of the control section moving upstream or some such idea, since we are speaking only of continuous

flow. Observation shows that the flow remains practically parallel down to a relatively short distance from the brink.

In order to determine the relative magnitude of the friction effect at the terminal section, an analysis of that effect was made by the authors on the basis of Eq. 6. Ignoring common factors, the ratio of the friction force to the accelerative force (per unit volume) equals the ratio of the quantity f/2 to the quantity ($Y\, d\beta/dx - \beta\, dy/dx$). That ratio was found to be always less than 4%.

Zone D - In this zone one further control is removed, namely, boundary friction. As the stability conditions for zone D and for zone C are different (albeit slightly), some discontinuity must take place at the terminal section. Evidently, no such discontinuity is possible in the central region of the flow (outside the boundary film). It must, therefore, be confined to the boundary region. Inspection of any photograph of a free overfall will reveal the existence of a separation line in the free jet emanating from the lip of the fall (see Fig. 2 in the original paper). Thus the free jet consists really of two jets, not one.

Owing to the relatively small thickness of the lower jet at the brink, the terminal depth would be influenced mainly by the conditions of stability for the upper one. These can depend only on the discharge and on the bed slope (provided the approach channel is sufficiently long). In the writers' experiments the ratio of the flume length to the terminal depth was of the order 40 for a horizontal bed and 50 for a sloping bed. That the flume length was adequate for the full establishment of the terminal depth is borne out by two facts. In the first place, the ratio Y_t/Y_c was found to be a constant for each bed slope within the range of discharges passed, as exemplified in Fig. 4. If the flume length were not sufficient, some variation in that ratio would have taken place as the discharge was increased. Secondly, the rate of convergence $(dy/dx)_t$ was also found to be constant for the same bed slope (Fig. 6). This implies a geometric similarity which would be realised only if the flow conditions at the terminal section were influenced solely by the requirements of stability in the free jet.

In the light of these facts, it will be seen why the ratio Y_t/Y_c should be practically independent of the frictional properties of the channel and also (provided the flume length is sufficient) of the upstream flow conditions. Incidentally, it may be mentioned that the writers had actually tested the effect of roughness of the channel bed on the terminal depth. The observations with a horizontal bed were repeated after coating the bottom of the flume with sand of a grain diameter varying between 0.5 and 2.0 mms. No appreciable difference in the terminal depth was detected.

Now we are in a position to obtain a better view of the question of negative pressures. In trying to overcome increasing bed friction, the flow will lose potential energy (which is the only source of energy it can draw upon) at an increasing rate. With a horizontal bed, the value of the pressure correction factor α, in the average for the range of discharges passed, was found in the writers' experiments to fall at an increasing rate from about 0.85 at a section 10 cms inside the brink to 0.20 at the brink. With a bed slope of 0.0275, the corresponding values were 0.94 and 0.05. It is clear that with such steep pressure gradients, the chances are that the pressure will somewhere fall below atmospheric.

Apart from the results of the writers' experiments, however, it is to be noted that the flow has greater freedom to gain momentum in the free jet than inside the channel, and spatial acceleration in the free jet must,

consequently, be greater. In the first stage of the free flow (just outside the brink) the rate of change of position energy is somewhat restricted by the fixed bed slope. Hence the rapid gain in kinetic energy is secured mainly at the expense of the pressure energy. An illustrative example may be cited in the case of a train rolling, with the brakes partly laid on, down a sloping track which ends at a precipice. As the carriages pass over the brink, the couplings between them will obviously be put in tension.

The indications of the potential flow theory carry very little weight in the present study. The application of that theory to the free overfall is possible only when the channel bed is horizontal and the initial (parallel flow) depth equals the critical depth. No general conclusions can be drawn under such limited conditions. There is, however, an interesting point in this connection which seems to have escaped notice so far. If the pressure were supposed continuous on the bottom stream-line (extended into the free jet) then the pressure within the brink section should be wholly negative. Applying Eq. 13 to a point on the channel bed just inside the brink, we find that $r_f = \infty$, $\cos \delta = 1$ and $dp/dn = -\gamma$. Since $p = 0$, it follows that higher up in the section the pressure must be negative.

As regards the manner of variation of the quantity $d\beta/dx$, to which reference has been made by Messrs. Carstens and Carter, the potential flow theory is even less helpful. In potential flow, it is to be noted, the energy constant must not only be the same for any streamline but also for all streamlines in the flow field. In other words, the flow is supposed to start with uniform energy distribution. In the flow of a viscous fluid, on the other hand, energy distribution is never uniform in the presence of a solid boundary, even when the flow is parallel. This, of course, is the result of non-uniform velocity distribution. In the case of a free overfall, there will be a tendency towards more uniform velocity distribution as the fall is approached so as to satisfy the conditions of stability in the free jet (where the boundary is eliminated). Thus the quantity $d\beta/dx$ is bound to be negative in the region of the fall. This has been confirmed experimentally.

It is hoped that the foregoing exposition of the problem will help to dispel the confusion that appears to exist on the subject and to pave the way for further investigation of secondary questions on the right lines.

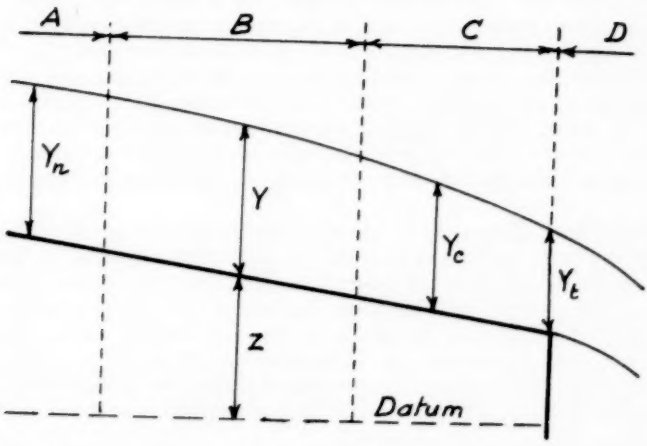

Fig. 8 - Drop-down Flow Zones.

Discussion of
"MECHANICS OF STREAMS WITH MOVABLE BEDS OF FINE SAND"

by Norman H. Brooks
(Proc. Paper 668)

JAMES R. BARTON.[1]—In the field of sedimentation where controlled and reliable data are rather scarce, it is encouraging to see new results added to the rather meager ones already available. The author's measurements appear to be fairly complete, and the care with which they were taken should make them quite reliable data.

Recent studies at Colorado A & M College, done for the Missouri River Division of the Corps of Engineers, indicate that most of the conclusions arrived at by the author are valid. Conclusions 4(a) and 4(d) agree very well with findings here at Colorado A & M. The changing pattern of bed roughness found here is also similar to that reported by the author.

The sediment used in the Colorado A & M studies had a median diameter of 0.180 m. m. and a standard deviation of 0.037 m. m. The bed roughness was found to vary rather systematically with the mean velocity of flow as shown in Fig. I. For velocities below 1.8 ft./sec. the bed was covered with dunes similar to those shown in Fig. 5. Between a velocity of 1.8 ft./sec. and 2.4 ft./sec., sandbars occurred on the bed, and the water surface was undulating so that uniform flow could not be established. The flume used was 4 feet wide and 70 feet long, and the sandbars were generally 6 to 10 ft. in length, and the crest of the sandbars was always diagonal to the direction of flow. Each sandbar generally extended across the entire width of the flume. This sandbar phase seems to coincide with the author's meandering bed condition. For velocities beyond 2.4 ft./sec., the sandbars were swept away and the bed became free of all dunes and ripples. This phase has been named by the writer as a plane bed, as contrasted to the author's smooth bed. The term smooth was avoided because, although no ripples of any size existed on the bed, the sand grain roughness was sufficiently large so that the bed was not hydraulically smooth. Calculation of the thickness of the laminar sublayer indicated that the medium grain diameter and the laminar sublayer were approximately the same size, leaving many of the larger grains projecting above the laminar sublayer. Further increase in velocity beyond 4.0 ft./sec. resulted in the formation of antidunes although no data were gathered for the antidune condition.

During many of the experiments, the depth did vary during the period of establishment of uniform flow. The variation of depth was often caused by non-uniform movement of the sediment through the system. However, after the flow reached a stable uniform stage, the depth also became stable.

The author seems to have gone out on a limb in arriving at conclusion 4(b). In fact the writer is left a bit confused from conclusion 4(b) after reading conclusion (3). The confusion probably arises from the fact that slope is not mentioned in (3) while it is a definite part of 4(b), and so it is hard to visualize the

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exact interrelationship. However, under any circumstances, data gathered here at Colorado A & M do not substantiate conclusion 4(b). Although the writer will make no attempt to explain conclusion 4(b), he does feel that more data from several different flumes should be gathered and analyzed before 4(b) should be accepted as a reality. Since no other experimenters seem to have reported such a finding, it may be a characteristic of the flume used which produced such unusual results.

In the 4-foot flume at Colorado A & M, recent data tend to challenge the conclusion that, for a given slope and discharge, two depths are possible. One series of tests is presented here in table form:

Discharge Q c.f.s.	Mean Depth ft.	Average velocity V = Q/A ft./sec.	Manning n	Slope	Bed Condition
5.8	0.94	1.54	0.0216	0.00056	Dunes
5.8	0.85	1.70	0.0204	0.00068	Dunes
5.8	0.79	1.84	0.0183	0.00071	Sandbars
5.8	0.66	2.20	0.0154	0.00090	Sandbars
5.8	0.61	2.38	0.0137	0.00093	Plane bed

The median diameter of the sand as 0.180 m. m. Each run was allowed to continue for 6 to 18 hours to assure that uniform conditions existed, and invariably, if the depth changed, the slope also changed. The slope and discharge could not be held constant and have the depth vary. Additional data are recorded in Fig. II, which shows discharge plotted against depth with slope as the 3rd variable. It seems logical to assume that if more than one depth can exist for a given slope and discharge, the data should exhibit more scatter than the normal scatter pattern shown in Fig. II. Some of the scatter may be explained by a variation in temperature. The temperature varied from 15° C to 23° C for Curve A, with most of the runs being made at a temperature of 19° C. The extreme temperatures are shown next to the points involved. All runs for curve B were run at temperatures between 20° C and 22° C except one, which was 27° C, as shown on the graph.

Observations of the experiments and analysis of the data from the experiments made at Colorado A & M College both fail to support the author's conclusion 4(b). Therefore, the writer feels that conclusions (1) and 4(b) should not be generally accepted until further evidence obtained from more than one experimental source has verified the same findings as those of the author.

H. A. EINSTEIN,[1] M. ASCE, AND NING CHIEN,[2] A.M. ASCE.—The author has presented some very interesting results in his paper, based on which he expressed some doubts on the generality of existing methods in describing the roughness and sediment transport of alluvial channels. According to his findings, the transportation rate could not always be expressed as a unique function of the bed shear-stress, the channel geometry, and properties of the sediment, because of the extreme variability of the channel roughness. The modern theories on sediment transport[7],(A) have fully recognized the important role played by the bar resistance in defining the alluvial channel flow. At low and medium rates of bed-load motion, the bar resistance is a major

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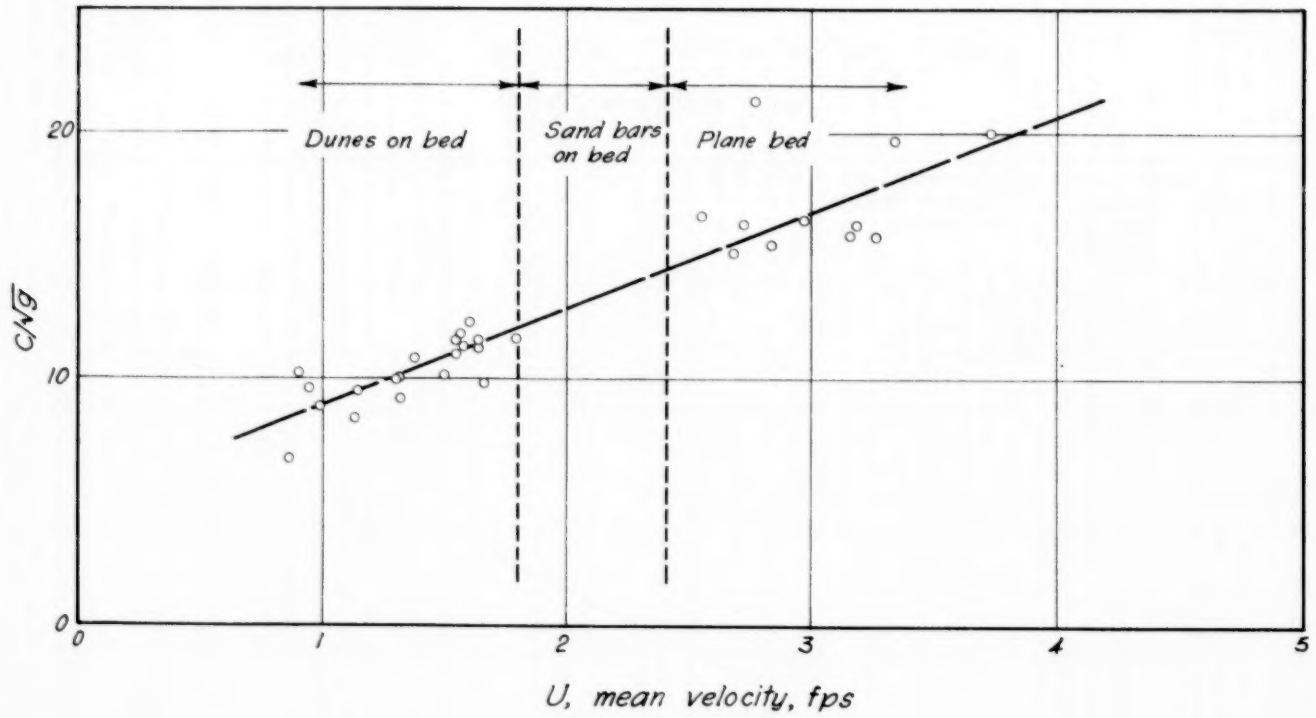
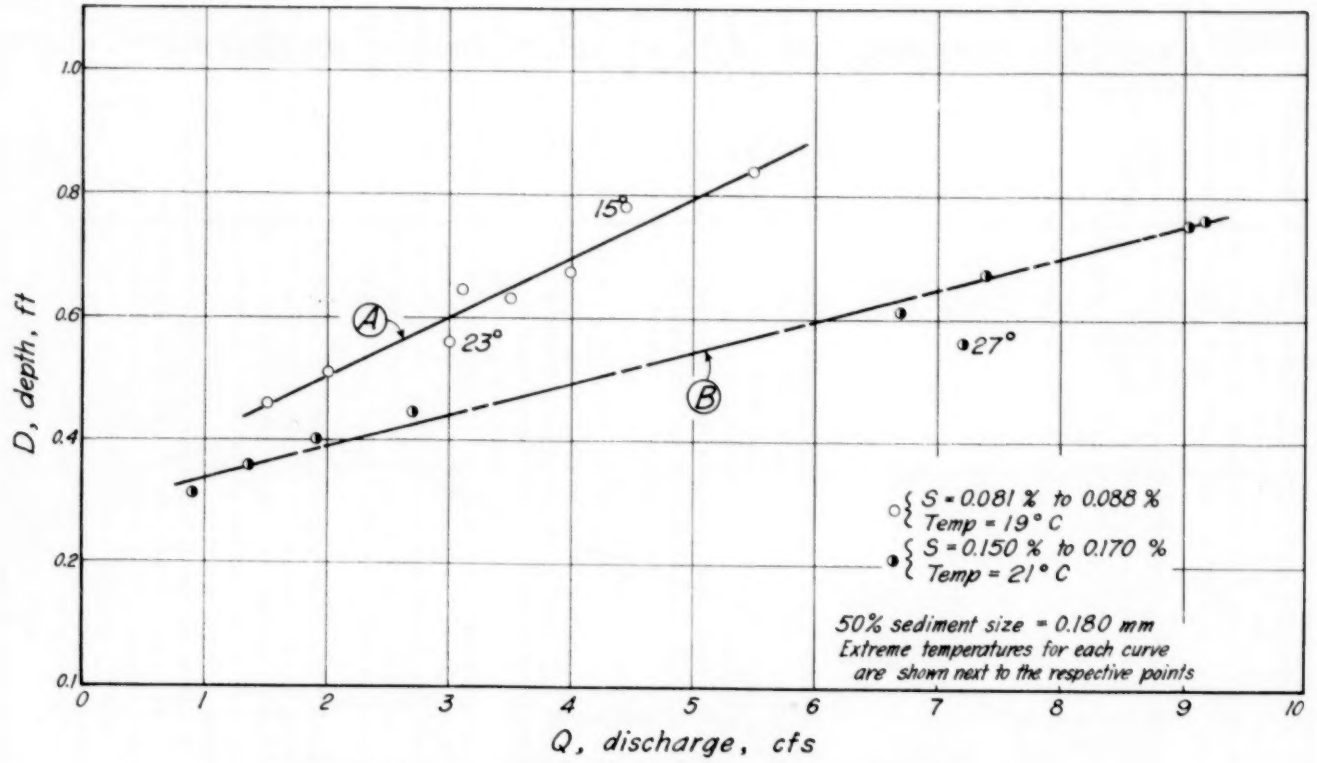


Relationship between Mean Velocity and Roughness Coefficient

FIG. 1



Relationship between Discharge and Depth

FIG. 2

part of the total frictional resistance of fine sand beds, and yet the shear transmitted to the bed through the shape resistance of the sand bars has only a minor effect on the bed-load transport. Therefore, the sediment transport has been related only to the other part of the bed shear, namely, the grain resistance of the bed material. Thus the conventional methods permit unique determination for most flows of any two of the following four variables:

unit discharge, unit sediment load, depth, and slope

if the other two are known.(B) The author claimed that although the depth and slope of the alluvial channel are uniquely determined by the discharge and the sediment load, the reverse is not true. This is due to the fact that for the same depth, slope, and bed material, in one case the bar resistance may be large and sediment load and discharge small, and in another case the bed may become quite smooth and be able to carry a larger flow and sediment load. This discussion attempts to reassess the author's findings as follows:

1) It will be shown that the behavior of bar resistance in the author's experiment differs with the findings of past river observation, and it is precisely this difference which results in the complicated flow pattern as observed by the author in his flume;
2) it will be shown that some of the phenomena which the author has observed in his flume and which he introduces as new and as contradictory to past experience are actually in full agreement with previous theories. and are believed to apply to rivers within very limited ranges of flow and sediment conditions;
3) it will be shown that the field observations quoted by the author as proof of his findings do not fit the case which the author intends to illustrate, and can actually be fully explained by the past methods of description; and
4) it will be attempted to explain the deviations of author's result from the river data by an interpretation of the author's and other similar results in form of a bar resistance graph as proposed by the senior writer and Barbarossa.[14]

In 1951, a paper[14] was published indicating that the resistance of a movable bed can be divided into two additive parts, the first of which is related to the roughness of the surface as defined by the representative grain size in the bed. It is the roughness obtained if the grains are arranged in a perfectly regular manner so as to create a perfectly flat rough surface. An additional frictional effect is obtained if the same particles are arranged in a more irregular fashion such that they protrude irregularly from an average bed surface or if they actually arrange themselves in the form of bars or other irregularities. This latter part of the friction was shown in reference 14 to be a function of the bed-load transport rate, which itself is a function of the parameter $\psi' = \frac{\rho_s - \rho_f}{\rho_f} \frac{D_{35}}{R'_b S}$.

The functional relationship between v/u_*'' and ψ' is reproduced in Fig. 13 by the heavy curve N, where v is the average velocity and $U_*'' (= \sqrt{SR_b''g})$ is the shear velocity corresponding to the bar resistance. The author's data are next analyzed in terms of the same parameters, and plotted in Fig. 13 as open circles for the case D = 0.16 mm. It can be seen that at low stages the bars behave the same way in author's flume as that in rivers. At higher flows the experimental points begin to deviate systematically from the curve and line up along a higher curve, indicating that sand bars disappear sooner than in

natural streams. It will be shown later that if the bars behave this way in the author's flume, the flow relationships become extremely complicated indeed.

In order to demonstrate whether different flows and sediment loads can be carried by the same channel at equal slope, depth and bed material, the following argument will be conducted on the basis of graphs which show the various aspects of the relationship between the bar resistance and the flow intensity. Into Fig. 13, three hypothetical curves I, II, and III were introduced, each giving all possible combinations of Ψ' and v/u_*'' for a given combination of bed material size, slope, and depth, assuming no relationship to exist between Ψ' and v/u_*''. No bank friction is introduced in this derivation. The three curves correspond to the following values of the variables:

Curve No.	Bed Material Size, mm.	Slope	Depth R_b, ft.
I	0.16	0.0001	10
II	0.16	0.0001	14
III	0.16	0.0001	20

Curve I is, for instance, obtained as follows: for each point of the curve the depth is arbitrarily divided into R_b' and R_b'', for instance $R_b' = 6$ ft. and $R_b'' = 4$ ft. Now Ψ' can be calculated as

$$\Psi' = \frac{\rho_s - \rho_f}{\rho_f} \frac{D_{35}}{S R_b'}$$

and v/u_*'' is obtained, using the logarithmic friction formula (or any other such formula) for v, as

$$\frac{v}{u_*''} = \frac{5.75 \log_{10} (12.27 \times R_b'/D)}{\sqrt{R_b'' S g}}$$

The three curves represent, thus, all possible combinations for R_b' and R_b'' for the three depths. If curve N applies to these flows one may conclude that its intersection with the three curves gives the actual solutions. One would read thus from Fig. 13 that the 10 ft. depth has only one solution with $v/u_*'' = 10.3$, a well-defined bar condition. Curve III does not intersect curve N, indicating that such an intersection must occur at infinity—the bed is flat and bar resistance is negligible. Curve II, however, follows curve N for a short distance, indicating that any one of these points is a possible solution. This is a case where various discharges and sediment rates are possible at the same combination of depth, slope and grain size. Here, the old prediction method[14] shows the exact behavior which Brooks claims to have newly discovered. To further confirm this point, the stage-discharge curve for this particular channel is constructed according to the methods outlined in reference 14, and shown in Fig. 14. The bar resistance is the dominating part of the total frictional resistance at low stages, resulting in a slow increase of discharge with the rise of stage as shown by the part AB of the rating curve. After the depth reaches 13.5 ft., further increase in flow actually makes the bed smoother. That means an increase in R_b' is accompanied by a decrease of R_b''. The changes in both hydraulic radii are of the same order of magnitude, resulting

in an almost constant water depth of 14 ft. in that range of flows (part BC of Fig. 14). At still higher stages, the bar resistance becomes insignificant and the discharge increases rapidly with the stage, as shown by part CD of the rating curve. It is immediately evident that any attempt to correlate the measured discharge and sediment load with the channel characteristics in range BC of the flows will become a hopeless task, considering the uncertainties and errors involved in field measurements. The existence of such uncertainty in the prediction of discharge and sediment load in very limited ranges of conditions was known previously, but not much significance was attached to this fact because of the rather rare occurrences of these cases. Thus, the statement that more than one solution of discharge and bed-load rate is possible for a given depth, slope, and grain size does not in itself prove that the old approach to the determination of channel roughness is not applicable to this case.

For a better understanding of the author's findings, curve IV, which is similar to curves I to III, is constructed in Fig. 13 for the conditions existing in the first part of the author's experiment. It represents a hypothetical bar resistance curve if various flows and loads are possible in the flume at a depth of 0.245 ft. and at a slope of 0.0025. Over a large range of conditions the line follows the experimental points, justifying the author's claim that in his experiment the mean velocity and sediment concentration are not uniquely determined by the channel and flow geometry. A further analysis of the experimental data reveals that the results are even more complicated than that. If the same depth and slope can carry different discharges, the question arises as to whether the same discharge can be carried by different depths at a given slope. To ascertain this point several hypothetical bar resistance curves are drawn in Fig. 15 for both natural streams and conditions existing in the author's experiment. Each of these lines represents in this case the bar resistance relationship if the same discharge can be carried by different depths at the same slope. For natural streams such a condition is not possible, as each of the hypothetical lines intercepts the actual bar resistance curve only at one point. And yet it is quite possible in the author's flume, as part of the hypothetical line again goes through several experimental points. Indeed if the author's flume study is conducted in such a way that a rating curve with reasonable range can be obtained, it will follow either of the several possibilities as shown diagrammatically in Fig. 16. Or, it may even follow one type of curve at a certain range of stages, and another type of curve at other stages. Among those various possibilities, in one extreme is the curve type A which, as shown in Fig. 14, is also sometimes possible in natural streams. In another extreme is the curve type D which occurs rarely, if ever, in nature. Operating under such intricate circumstances, it is no wonder that the author's data scatter all over the place in Figs. 6 and 7!

The author has quoted several river observations to demonstrate that what he has found in his flume also happens in nature. These field measurements seem to indicate that for the same discharge and slope, the depth changes with the sediment load. The author also implied that for these examples no change in the bed material has taken place. The present knowledge on the behavior of alluvial channels indicates that the depth and unit sediment load are uniquely determined by the slope and unit discharge. The apparent contradiction between these two statements must be reconciled either by accepting the argument that rivers do behave as suggested by the author, or by showing that either slope, discharge or grain size actually changed sufficiently to produce a changed depth and sediment load.

Most of the river data quoted in the paper are taken from the work of Loepold and Maddock[10,11] in which the characteristics of the channel, the discharge, and the suspended load are correlated quantitatively. Fully recognizing the difficulties involved in sampling the bed-load, the analysis, nevertheless, suffered by the fact that the sediment load used is not only the bed material load which really controls the stability of the channel. It has been demonstrated[C] conclusively that under the same flow conditions, the transport rate of wash-load may change drastically, depending upon the availability of the material from the upstream watershed. During the dry period of the year, much of the sedimentary material is made available in the watershed by weathering or other processes. The first rain or snow melt erodes these loose materials accumulated at the ground surface, bringing a large amount of sediment into the stream channels. After the loose material is removed from the surface, the erosion caused by the surface runoff is drastically reduced. This accounts for the fact that in the Colorado River at Grand Canyon, Arizona, the suspended sediment load, in which the wash load constitutes the major part, is ten times larger in the rising stage than that in the falling stage at a comparable discharge. Indeed even if the change in suspended load actually reflects a change in bed material load, the river will not respond to this change by filling up or scouring the bed to a depth as much as 8 ft. in a matter of days. What actually happened in the Colorado River at Grand Canyon for that particular occasion was a transient change of the local river bed during a flood, as pointed out by Lane and Borland.[D] It should not be interpreted as an adjustment of river regimen in response to the change of independent variables imposed on the channel by the watershed.

The example of the Colorado River at Yuma, Arizona, on the other hand, does reflect a change in river regimen after the completion of a series of dams (Hoover Dam in 1935, Parker Dam and Imperial Dam in 1938) along the upstream channel. The data indicate that for the same slope and for a typical flow of 12,000 cfs, the depth jumped from about 5 to 9 feet, the velocity decreased from about 5 ft/sec. to 3 ft/sec., and the Manning n increased from 0.013 to 0.030, due to the reduction of incoming sediment load. The pertinent point in interpreting the data revolves around the question of whether there is a change in bed material size caused by the upstream dams. Immediately after the closure of a dam, the clear water released by the dam will erode the downstream channel, causing a rapid degradation immediately below the dam.[E] The sediment removed from the bed by clear water consists, to a large proportion, of fine particles, causing the bed material to become gradually coarser. As the bed degrades, gravel strata or clay lense, which were covered before, become exposed to the flow. These two types of material may actually prevent excessive degradation. A detailed mapping of the bed material of the Missouri River downstream from Fort Peck Dam[F] indicates the exposure of such gravel lenses and clay strata which are scattered along the channel. However, at the places where the underlying layers are not exposed, the change of the bed material size may remain small. The measurements made in the Colorado River[G] also indicate that the bed material size increased from 0.175 mm. to 33 mm. at a distance of 12.8 miles below Hoover Dam, and from 0.18 mm. to 7.0 mm. 17.4 miles below Parker Dam. Both changes took place in a period of thirteen years. The measuring section at Yuma, Arizona, is located 15.8 miles downstream of Imperial Dam. Between January, 1940, and September, 1951, the total volume of bed material removed between Imperial Dam and mile 15.6 is estimated to be sixty million cubic yards. One should expect that with the degradation of the channel bed, a coarsening of

bed material in the downstream channel took place, even if Imperial Dam is of much smaller capacity than Hoover and Parker Dams. A survey of the bed-material samples taken at Yuma[G] indicates that the size varies from year to year in a rather unsystematic manner. The following extreme values are picked out from the data:

D_{50} = 0.125 mm. in 3-24-40
0.140 mm. in 11-8-51
0.320 mm. in 1-12-46
0.390 mm. in 4-13-47

and hydraulic calculations[7] are made for the two cases, taking the representative bed material size as 0.14 mm. and 0.35 mm., respectively. Table I gives the result of the calculations. The agreement between the measured and the calculated values is excellent. It is rather surprising that a small change in bed material size results in such a significant change in Manning's n. This is due to the fact that as the bed material becomes coarser, the same flow is only able to move a much smaller load, resulting in a much larger bed undulation.

The soundings in the Missouri River near Omaha, as cited by the author, revealed that the channel bed is deeper and rougher at one side, and shallower and smoother at the remaining part. From June to August, 1951, when the Missouri River was flowing at a constant discharge of about 65,000 cfs, a large number of bed samples were taken by the Corps of Engineers, the mechanical analysis of which indicated that the average bed-material size is 0.15 mm. in the flat area and 0.24 mm. in the rough area. And the erratic behavior of the Missouri River bed can be explained, at least qualitatively, by the difference in the material which composes that part of the bed.

Knowing that the behaviors of natural streams can be fully explained by the conventional methods without resorting to any drastic new interpretations as proposed by the author, one must ask the question of why the observations made by the author in his flume have never before been revealed. The writers, after reading the author's voluminous thesis,[5] are very much impressed by the author's carefulness and thoroughness in conducting his experiments. It is extremely unlikely that the data may contain any serious errors in measurement or in interpretation. In order to find an explanation of the results, a survey of available data on other flume studies was made and showed that there exist at least two other sets of measurements which also used fine sand with comparable flows. The first one was conducted by the senior writer in 1944-46 at the California Institute of Technology, and a more recent one is published by Barton and Lin[H] at Colorado A and M. The range of the main variables used in these studies are listed in Table II. The data are analyzed again in terms of the bar resistance, v/u_*'', and the flow intensity, Ψ', and are plotted in Fig. 17. Most interesting is the fact that the results of each flume study are consistent in themselves, and yet deviate from the river curve and from the other curves. In both the experiments by Brooks and by Barton and Lin, the sand bars disappear sooner than that in nature. However, this happens at different flow intensities for the different series. Proceeding from low to high flows, the beginning of deviation from the river curve follows the following sequence:

Brooks, $D = 0.16$ mm., $S_o = 1.075$
Brooks, $D = 0.10$ mm., $S_o = 1.11$
Barton and Lin, $D = 0.18$ mm., $S_o = 1.17$

The results obtained by the senior writer, although limited in range, fall more or less along the river curve even at comparatively small values of Ψ', and have a sorting coefficient S_0 = 1.19 - 1.22. The difference in the behavior of the bar resistance in these studies must reflect in one way or another the difference in conditions employed in these experiments. As far as the writers are able to ascertain (see Table II), the only significant parameter seems to be the sorting coefficient of the bed material. Up to a sorting coefficient of about 1.2, the more uniform the bed material is, the less persistent are the sand bars. A conclusion like this is not altogether unexpected. Local segregation of bed material takes place if the gradation is large. Fig. 18 is the sketch of such a bar. At the crest of a sand bar where the velocity is higher, the material is usually finer; and the coarsest is found at the trough. The changes in velocity and grain size seem to compensate each other, resulting in a more uniform shear over the bed than in the case of uniform bed material. Under such circumstances the sand bars are naturally more stable for mixtures with higher sorting coefficient. The argument as depicted above can easily be proved or disproved by running flume studies with absolute uniform material and by observing the behavior of the sand bars. If the uniformity of the bed material is indeed responsible for the deviation of the author's bar resistance from the river curve, one must not expect the conclusions determined from his study to apply to natural streams where the sorting coefficient S_0 of bed material is usually much larger than that of the author's material, as shown in Table III.

In concluding the discussion, the writers wish to present in Fig. 19 points showing the $\phi_* - \Psi_*$ relationship derived from author's experiments as compared with the theoretical curve proposed by the senior writer.[7] In spite of the great variability of the channel roughness in author's experiment, the agreement is quite satisfactory except the results of three runs. This again indicates that the concept in linking the bed-load transport with the grain resistance of the bed material only is basically sound.

ACKNOWLEDGMENT

The analysis made in preparing this discussion represents part of the research carried out for the Corps of Engineers, Missouri River Division, Omaha, Nebraska, under contract with the University of California.

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B. "Graphic Design of Alluvial Channels," by Ning Chien, Proceedings, ASCE, Vol. 81, Separate No. 611, Feb., 1955.

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D. "River-Bed Scour During Floods," by E. W. Lane and W. M. Borland, Trans., ASCE, Vol. 119, 1954, pp. 1069-1089.

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(NOTE: References are continued on page 27.)

Table I

Change in Regimen of Colorado River at Yuma, Arizona After the Closure of Imperial Dam As Indicated By Calculations and By Measurements

Time with Respect to the Closure of the Dam	Data Used in the Calculation				Comparison between calculated and measured results					
	Q cfs	B ft.	S ft/mile	D mm.	d in ft.		v in ft/sec.		n	
					Calc.	Meas.	Calc.	Meas.	Calc.	Meas.
Before	12,000	400	1.27	0.14	5.06	5.00	4.91	5.20	0.0139	0 0125
After	12,000	430	1.50	0.35	8.36	8.70	3.37	3.10	0.0306	0.0310

Table II

Range of Variables Used in Three Flume Studies

Investigators	Flume		Sediment			Flow		
	Width ft.	Length ft.	D mm.	S_o*	Transport Rate lb/sec/ft.	d ft.	v ft/sec.	S_e
Brooks	0.875	40	0.16	1.075	0.03-0.075	0.15-0.30	0.93-2.15	0.0018-0.0039
			0.10	1.110	0.0025-0.15	0.17-0.28	0.83-2.13	0.0013-0.0035
Barton and Lin	4	70	0.18	1.17	0.0006-0.45	0.30-1.38	0.90-3.74	0.00044-0.0021
Einstein	Same as in Brook's exp.		0.106--0.120	1.19-1.22	0.115- 1.34	0.43-0.44	1.76-3.13	0.0017-0.0039

*S_o = sorting coefficient = $\sqrt{\frac{D_{75}}{D_{25}}}$, unity for uniform material.

Table III

Sorting of Bed Material of Natural Streams

Stream	Location	Sorting Coefficient	Stream	Location	Sorting Coefficient
Brandywine Creek	Georgia, Pa.	3.02	Middle Loup River	Dunning, Neb.	1.40
Waar River	Holland	2.48	Boise River	Notus, Idaho	1.37
Brandywine Creek	Chadas Ford, Pa.	2.08	Niobrara River	Cody, Neb.	1.36
Cherry Creek	Denver, Colorado	1.73	Rio Grande River	San Acacia, N. M.	1.36
Upper Danube	Gyor, Hungary	1.73	Missouri River	Fort Randall	1.33
Mountain Creek	Greenville, S. C.	1.57	Bear River	Western Bridge, Utah	1.28
Salinas River	San Lucas, Calif.	1.57	Bear River	Trenton Bridge, Utah	1.24
Missouri River	Omaha, Neb.	1.56	Fraser River	Canada	1.24
Rhine River	Brugger Bridge	1.53	Tiber River	Rome	1.23
Colorado River	Yuma, Ariz.	1.49	Mississippi River	Baton Rouge	1.23
Rio Grande River	Bernardo, N. M.	1.47	Atchafalaya River	Simmersport-Melville	1.17
Mississippi River	Near Old River	1.45			
Rio Grande River	Bernalillo, N. M.	1.43			
Atchafalaya River	Barbre Ldg-Knotz Springs	1.42			

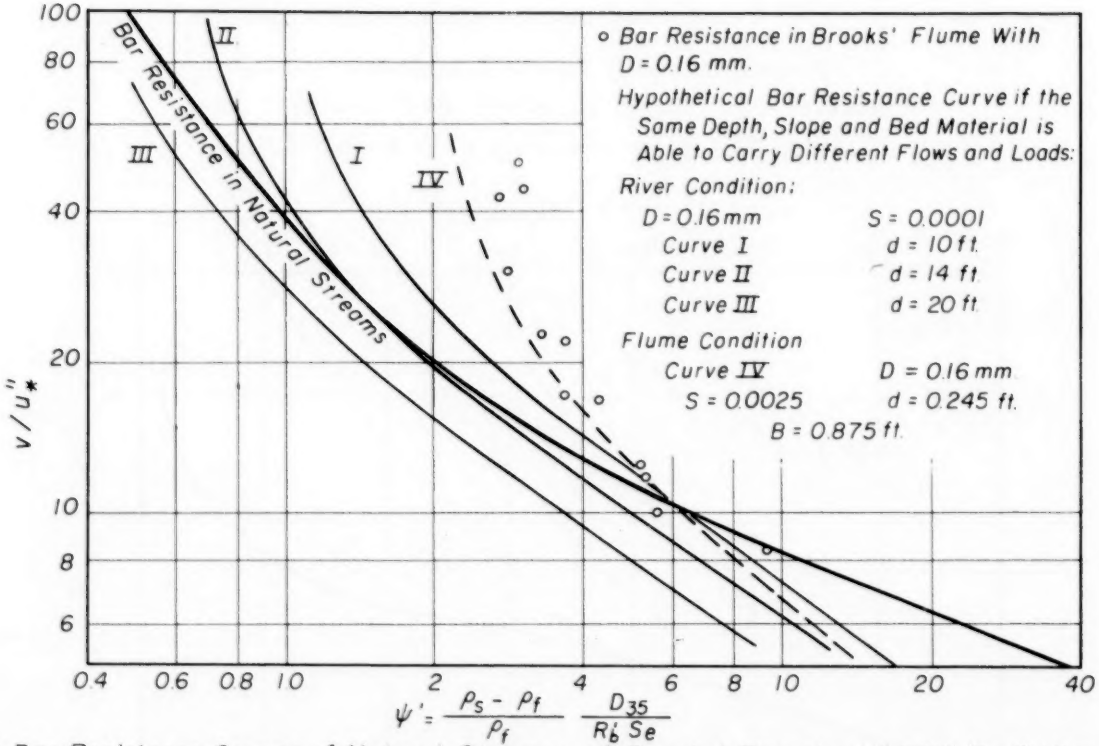

Fig 13 Bar Resistance Curves of Natural Streams, of Brooks' Experiment With D = 0.16 mm., and of Various Hypothetical Cases Where the Same Depth, Slope and Bed Material is Assumed to be Able to Carry Different Flows and Loads.

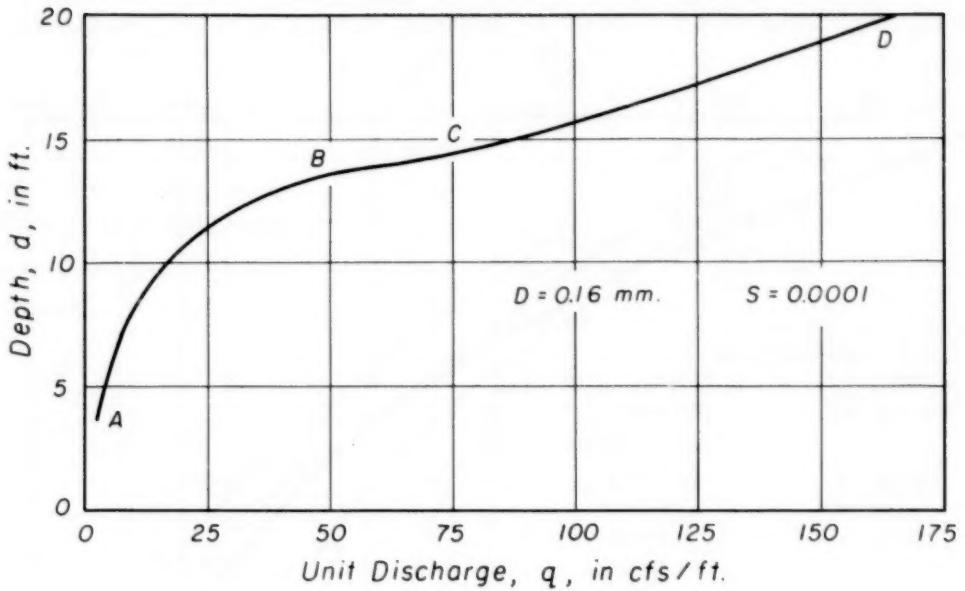

Fig. 14 Rating Curve of a Channel With Bed Material Size of 0.16mm. and Slope of 0.0001, Calculated According to the Einstein Method.

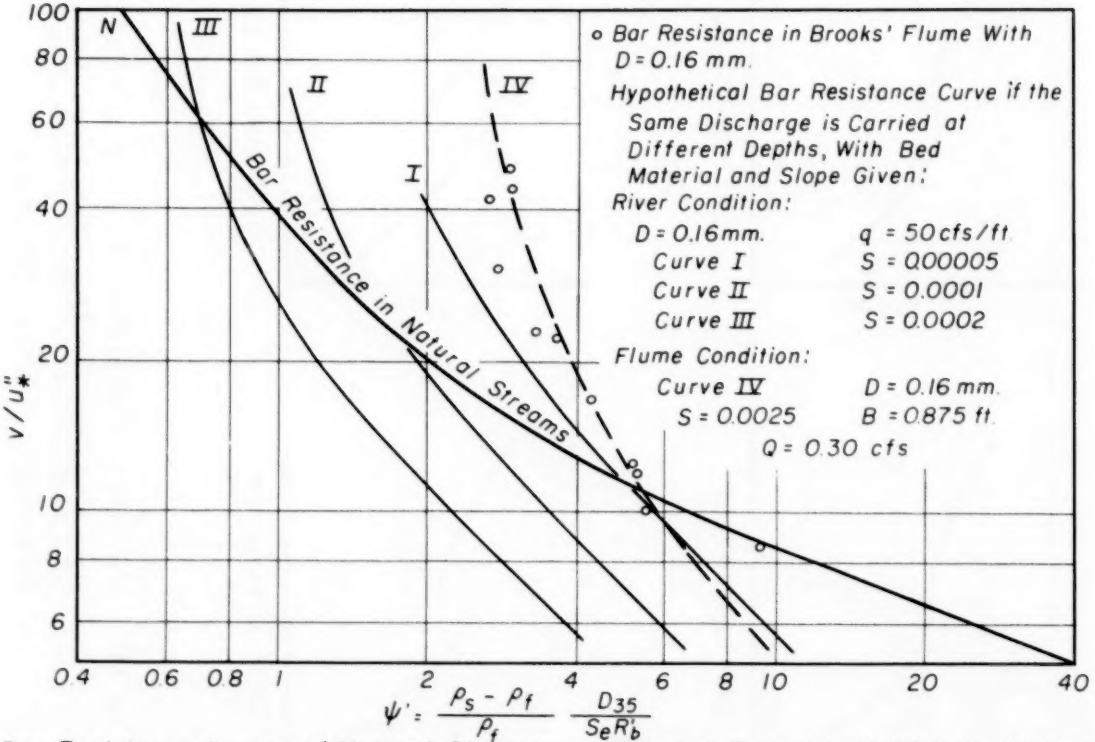

Fig. 15 Bar Resistance Curves of Natural Streams, of Brooks' Experiment With D = 0.16 mm., and of Various Hypothetical Cases Where the Same Discharge is Assumed to be Carried at Different Depths, With Bed Material and Slope Given.

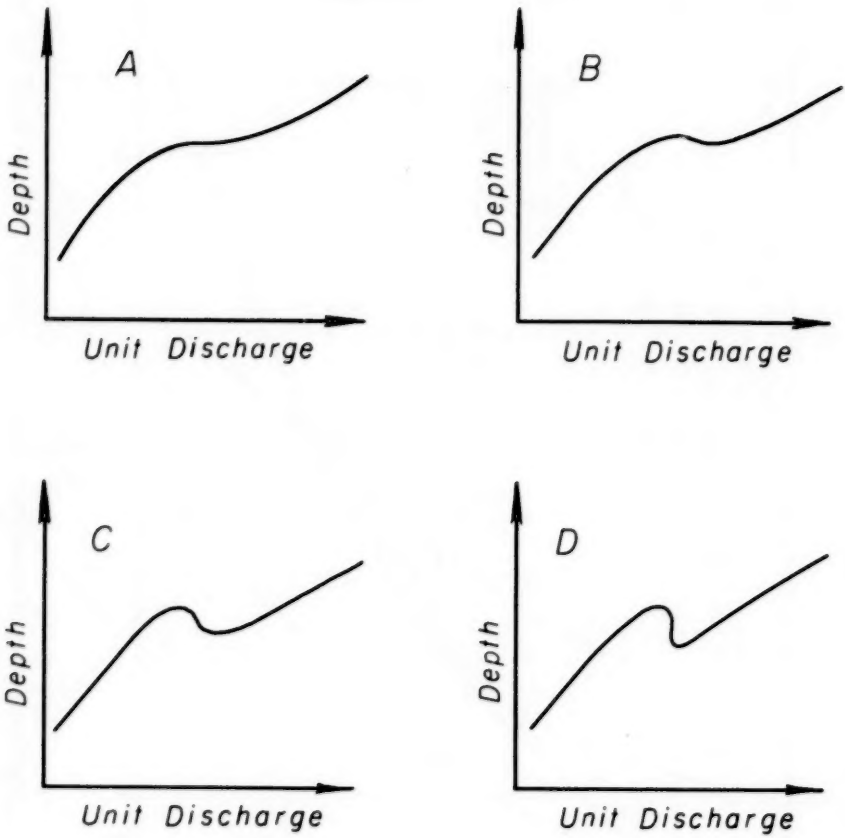

Fig. 16 Various Possible Rating Curves According to the Results of Brooks' Flume Studies

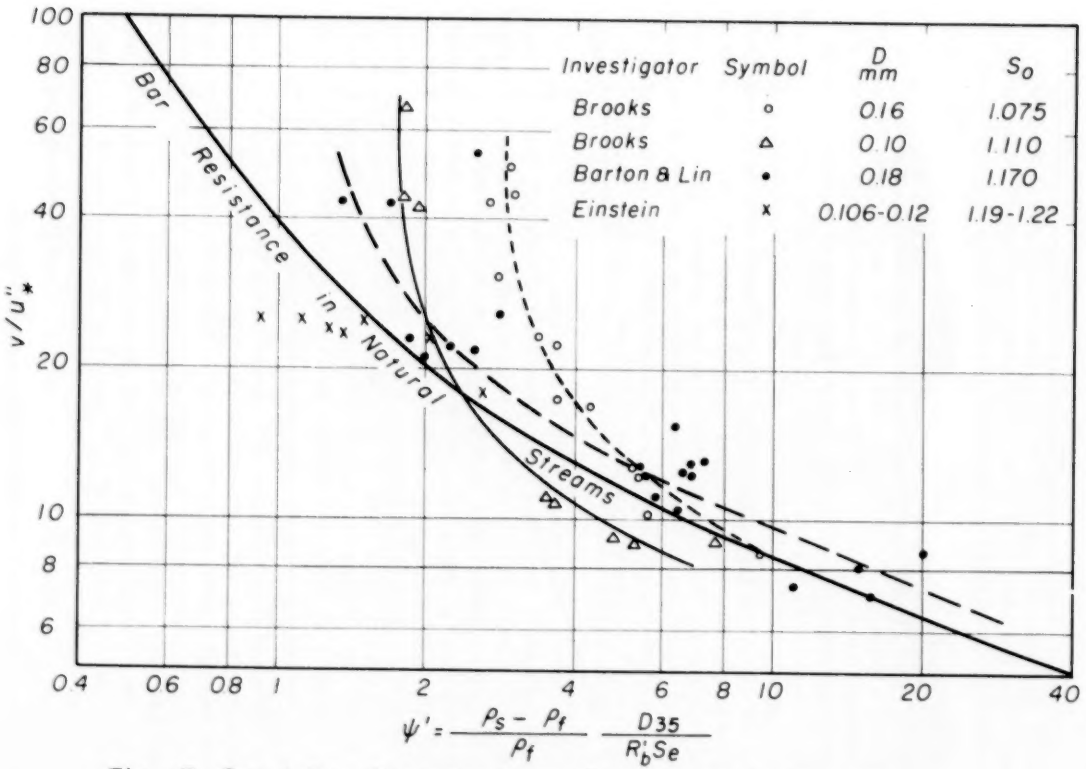
Investigator Symbol D mm S_o
Brooks 0.16 1.075
Brooks 0.10 1.110
Barton & Lin 0.18 1.170
Einstein 0.106-0.12 1.19-1.22
Bar Resistance in Natural Streams
v/u_*''
$\psi' = \frac{\rho_s - \rho_f}{\rho_f} \frac{D35}{R_b' Se}$

Fig. 17 Sand Bar Resistance in Flumes as Compared With That in Natural Streams

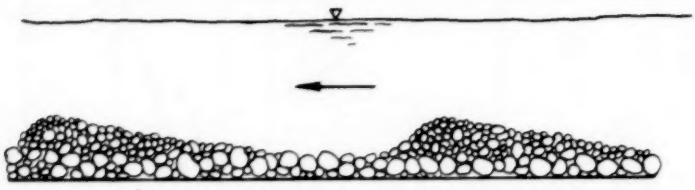

Fig. 18 Sketch of the Bed Segregation if the Gradation of the Bed Material is Large.

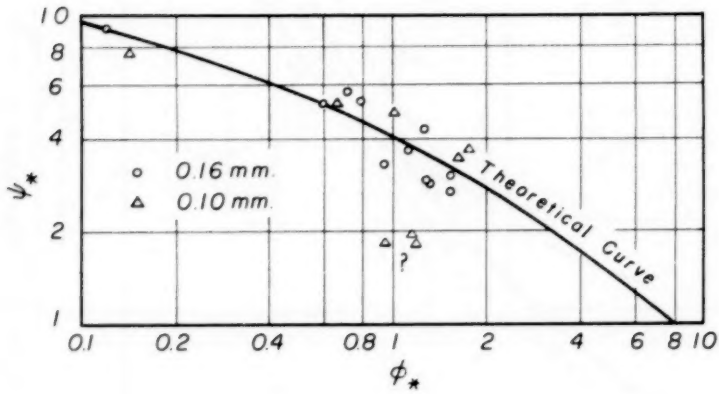

Fig. 19 Comparison of Brooks' Results With Einstein ϕ_* - ψ_* Relationship.

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G. "River Control Work and Investigations, Lower Colorado River Basin, Calendar Years 1950 and 1951," Office of River Control, Bureau of Reclamation.

H. "A Study of the Sediment Transport in Alluvial Channels," by James R. Barton and Pin-Nam Lin, Report No. 55JRB2, prepared for Corps of Engineers, Missouri River Division, Omaha; Colorado A and M (publication pending).

E. J. CARLSON,[1] M. ASCE, AND M. GAMAL MOSTAFA,[2] A.M. ASCE.— Some of the conclusions drawn by the author from his limited experimental data seem to contradict well-established theories of sediment hydraulics.

Several of the runs gave higher mean velocities or higher sediment transport for lower boundary shears, and from these runs the author concluded that it is impossible to take the shear or the slope as independent variables. What these runs may actually show is the known fact that when there is sediment in transport, the rate of energy dissipation does not depend only upon rate of flow, water section, and sediment size, but also upon the rate of sediment motion and the mode of bed formation both of which are, in turn, dependent upon the hydraulic conditions and sediment characteristics.

In an attempt to analyze the presented data, two points of interest are raised:

1. The total weight of sand used in the flume was 145 pounds which is extremely small, giving a depth of sediment on the flume bottom of approximately 1/2 inch. The author mentions the possibility of such being the reason that fully developed dunes did not change appreciably in height. This could mean that for some runs, due to shortage of sand, the friction was forced to be less than what would naturally occur if the sand were say 2 or 3 inches deep. Ten runs out of the 22 given in the data are reported to have shown sand dunes. For some, if not all, of these runs, one cannot be certain about the validity of deriving any conclusions as regards the relation of discharge to depth and slope, since the friction was more or less restricted.

2. The total weight of sand in motion was of the order of 0 to 5 percent of the total weight of sand in the flume. The sand particles in the bed were not of uniform size but had a certain gradation, as shown by the mechanical analysis in the author's Figure 3. The particles which were picked up by the flow and particularly after dunes had formed were certainly the smallest in size and, therefore, should not have been represented by the sedimentation mean of the bed. Determining the characteristics of the sediment in motion in each run by analyzing the dry sand from the samples that were taken for determining concentration would probably show an increase in mean size with increase of sediment load.

In spite of the two above-mentioned objections, the author's data should still show some trend if plotted non-dimensionally, save for the runs where friction may have been restricted through the exposure of a part of the flume bottom, because all the variables of flow, rate of transport of sediment, bed

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formation, and characteristics of both sediment in motion and in the bed should all be interdependent.

Plotting G/60 Q against slope, in the writers' Figure 1, different functions for different sand beds would be expected; however, the data has a wide scatter. Now if in every run, a sample of the sediment in motion was analyzed, each run would have certainly yielded a different mean size always lower than the sedimentation mean since the amount of sediment in motion is relatively very small and would necessarily be finer than the average bed material. Data plotted on the writers' Figures 1 and 2 would probably fit a group of lines for different sediment sizes if the actual size of the sediment in motion were known in each case.

In the writers' Figure 2, $\frac{r_b}{D}$ is plotted against G/60 Q both being nondimensional ratios. All runs giving dunes follow a certain function while those giving a smooth bed follow another function with the runs having so-called meanders appearing in both functions. It follows that for the same hydraulic conditions as given by r_b, S and Q, two different ratios of sediment to water discharge might have been produced one with a smooth bed and the other time with a dune formation. Yet it is doubtful that this could have actually resulted if every run was carried to sediment equilibrium condition under a uniform flow of water.

If a run was started with a small depth of flow and then the depth was gradually increased, dunes which form at small depths might not have had enough time to be removed. On the other hand, if a run was started with large depth of water and a smooth bed and then the depth was gradually decreased, dunes might not have had enough time to develop. Nonuniformity of the flow, unstability of flow conditions, or nonequilibrium of sediment discharges, or any of these factors combined would necessarily produce errors that seriously affect the results especially when measurements are in such a narrow flume and within such a small range of slope and depth variation.

HSIN-KUAN LIU.[1]—The author made some very interesting observations in his study and drew certain conclusions which are important if proved to be true.

The method of determining the flow depth used by the author is questionable, although no better method is known. The true effective depth of flow is not known because of the excessive dune movement, which affects the velocity distribution and channel roughness. Whether the author let the suspended material deposit or let the flow wash the bed when he stopped the flow was not mentioned. It seems that either one of these two cases must have occurred as he stopped the flow. Consequently, the measured depth could not accurately represent the true depth of flow—a high concentration of suspended load would result in heavy deposition thereby reducing the flow depth more than a low concentration of suspended load.

The author limited the flow depth to a maximum of 0.3 ft. on account of the narrow flume. Such a small depth in comparison with the large height of sand dunes (for example: 0.3' flow depth, and 0.12' height of sand dunes) makes one wonder if the average depth, as determined by the author, had any significance. In a flume 40' long by 10" wide, 145 lbs. of sand seems to be inadequate, for since the maximum dune height could reach 1.5",it was possible that the 1/2" sand layer could be entirely removed in some places, leaving the bottom

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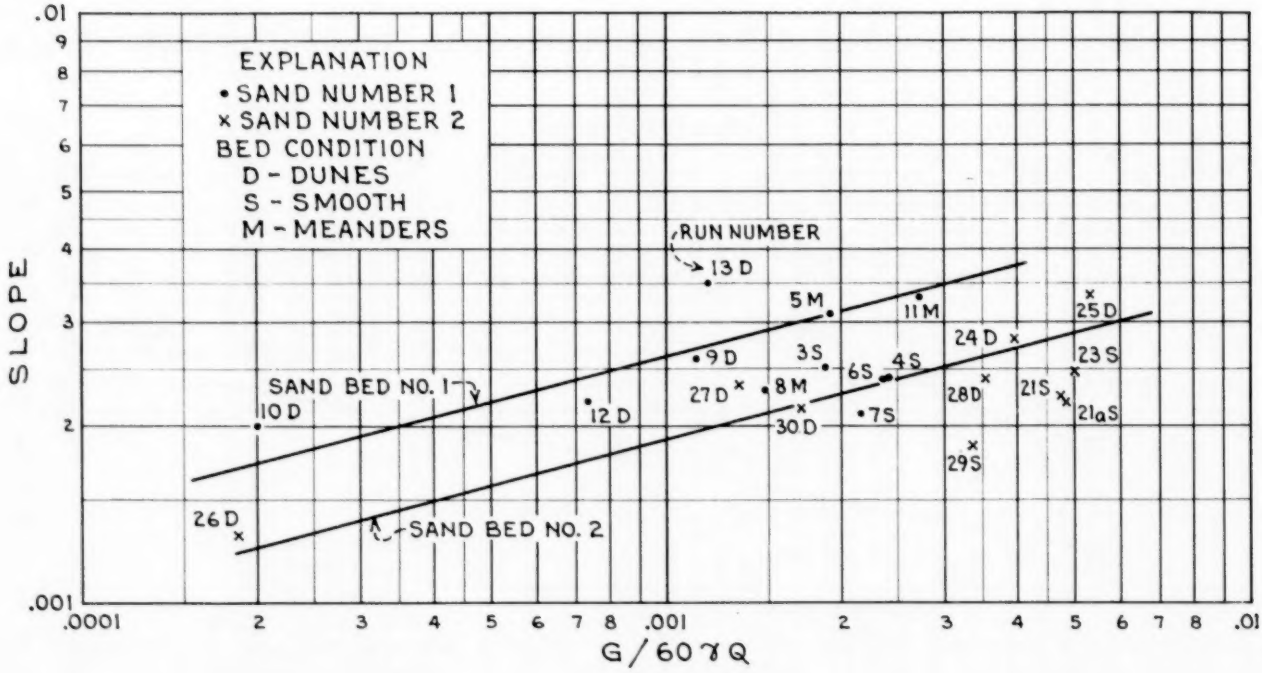

FIG. 1 - EFFECT OF SLOPE UPON RATIO OF SEDIMENT TO FLOW DISCHARGE

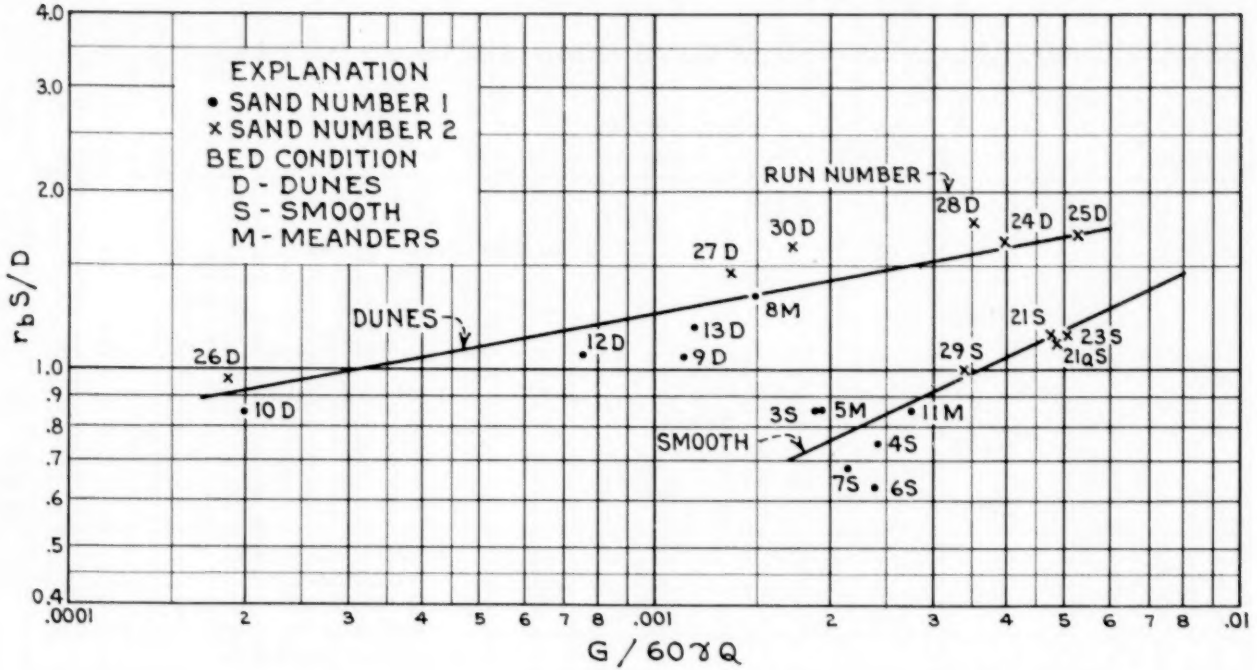

FIG. 2 - NONDIMENSIONAL PLOT SHOWING TREND OF DUNES AND SMOOTH BED

uncovered. How much this could affect the mechanics of sediment transport caused by the change of boundary conditions in such a case is uncertain, and therefore the question arises as to the justification of applying some of the author's conclusions to others' data.

Since no bed load sample was taken, it is doubtful that "the layer of sand covering the bottom of the flume contained practically all the sand in the system." The composition of the bed material may also affect the mechanism of sediment transportation.

When the flow is in equilibrium, not only the depth of flow and the pattern and distribution of sand on the bed remain constant, but also the energy gradient stays constant and equal to the water surface slope and bed surface slope. The well-known formula

$$T = \gamma RS \qquad (a)$$

is true only for the steady uniform flow conditions. Figure 2 indicates that the flow was not uniform, and the bed did not have a constant slope. For the same average flow depth in a reach, a converging flow acts differently from a diverging flow, neither of which can be represented by Eq (a).

To choose depth as an independent variable instead of using slope is favorable in the laboratory, which the author did. However, in a natural river, the bed slope is usually more or less a constant. It is the stage that fluctuates with and responds to the discharge more readily. Eventually, discharge, sediment load, bed slope, flow depth and roughness must be in equilibrium.

If discharge and sediment load are used as independent variables in two-dimensional flow; and flow depth, slope, and bed roughness as dependent variables, the following relationship exists for a given sediment

$$d = \phi_1(q, q_s) \qquad (b)$$

$$S = \phi_2(q, q_s) \qquad (c)$$

$$f_b = \phi_3(q, q_s) \qquad (d)$$

where q is unit flow discharge

q_s is unit sediment discharge

ϕ_1, ϕ_2 and ϕ_3 indicate functions

once Eqs (b) or (c) and (d) are known, slope or depth can be computed by using open channel flow formula and equation of continuity.

$$q = Vd = \sqrt{\frac{8g}{f_b}}\sqrt{dS}\cdot d$$

$$= \sqrt{\frac{8g}{f_b}}\, d^{3/2} S^{1/2} \qquad (e)$$

Eq (e) can be used to substitute either (b) or (c). Consequently there are only two major difficult problems in laboratory study of sediment transport: one is the sediment load formula, the other is the law of alluvial channel roughness. If Eqs (b) and (d) are single-valued functions, which the author found to be true, then according to Eq (e) there is only one value for S.

To sum up, there are five variables, q, q_s, d, S, f_b, of which q and q_s are chosen as independent variables; therefore, there are only three independent equations to express their relationship. If the equations are single-valued, the other three unknowns can be obtained by solving the three equations simultaneously.

Choosing d and s as independent variables in a laboratory flume to find the corresponding Q is a rather unwise procedure. Although it is not impossible, it is time consuming.

Take the rigid boundary open channel flow for example: it is easier to determine the normal depth of flow by letting certain discharge pass through a flume having a constant, uniform slope than to find the corresponding discharge for any predetermined normal depth and slope. It is even more difficult to adjust the flume slope experimentally for a given discharge and normal depth. Once the experimental relationship

$$d = \phi'(q, S, \text{boundary}) \qquad (f)$$

is found, any two variables, such as d and S or d and q, can be used as independent variables to find the third one.

The author found from his experiments that it was impossible to use d and S as independent variables because for given d and S, there were two or more equilibrium discharges. Such a statement is in contradiction to his other statement "f_b and $\bar{c}$ are uniquely determined by U and d," which means that the slope is also uniquely determined by U and d by using Eq (e). Then the question arises as to how the author could maintain two different equilibrium discharges by maintaining the same slope and depth, since slope is self adjustable to the discharge and depth of flow. The writer is concerned about whether the author waited long enough for true equilibrium conditions to be reached, especially since it was hard to do.

The author explained that it is possible to have two (or even more) different equilibrium discharges and sediment load because of variation of channel roughness due to dunes. Such an explanation is also doubtful because sand dunes do not constitute an independent variable. Although the determination of their characteristics and also channel roughness is still far from satisfactory, they must depend uniquely upon flow conditions. For a given flow condition, if it is assumed that there exists a unique value of channel roughness, it is impossible to have more than one equilibrium discharge.

Another possible explanation to the strange condition found by the author might be due to the particular experimental set-up used. It is noticeable that Run 3 and Run 9 did not have the same water surface condition, which may affect the flow condition for flow having 0.245' depth. More data from other independent tests are needed to verify the author's conclusions before they can be accepted.

The use of field data to verify laboratory conclusions needs careful examination, since, in general, the conditions are quite different. The sediment in a natural stream is seldom uniform in size distribution, consequently the sorting mechanism might be important. That the degradation rate slows down

below a dam as the years pass is due in part to the increase in bed material size by sorting.

Depth and velocity were favored by the author to be used as independent variables instead of d and S. Yet in some cases the author could not vary these two independent variables. It proves that independent variables are chosen with a view to facilitating the experimentation, but not because the variables are in themselves independent or dependent.

The writer wishes to make a few comments about the author's statement with respect to the sand dune phenomenon. From the writer's experiments on Ottawa 20-30 sand (1), bed load movement is in general in the form of sand wave motion, since ripples or sand waves appeared on the bed shortly after the beginning of bed load movement. The author's statement "when the total transportation rate was very small . . . , several layers of sand grains were almost always sliding along the bed," is doubtful. Before sand waves appear, the bed is plane, the bed roughness can be expressed by Manning's n, which is proportional to $d_{50}^{1/6}$. The corresponding value of f_b is usually smaller than those after the formation of sand waves. Therefore, it is not true that the largest f_b values are associated with the smallest sediment transportation rate.

The dune velocity measured by the author is plotted by the writer against the mean flow velocity as shown in Fig. (a). Data of $d_{50} = 0.1$ mm (2) indicate a similar relationship between dune velocity and mean flow velocity, as compared to the writer's experiments (1), the writer's data result in

$$C = 0.0008 V^5 \quad \text{for} \quad d_{50} = 0.690\,\text{mm}, \qquad (g)$$

the author's data result in

$$C = 0.000367 V^5 \quad \text{for} \quad d_{50} = 0.10\,\text{mm}. \qquad (h)$$

An approximate equation can be written for $d_{50} = 0.16$ mm as

$$C = 0.0006 U^5 \qquad (i)$$

where C is dune velocity in fps, U is the mean flow velocity in fps.

In conclusion, the writer wishes to point out that the author's conclusions (1), (2), (4b) and (4d) seem to be doubtful.

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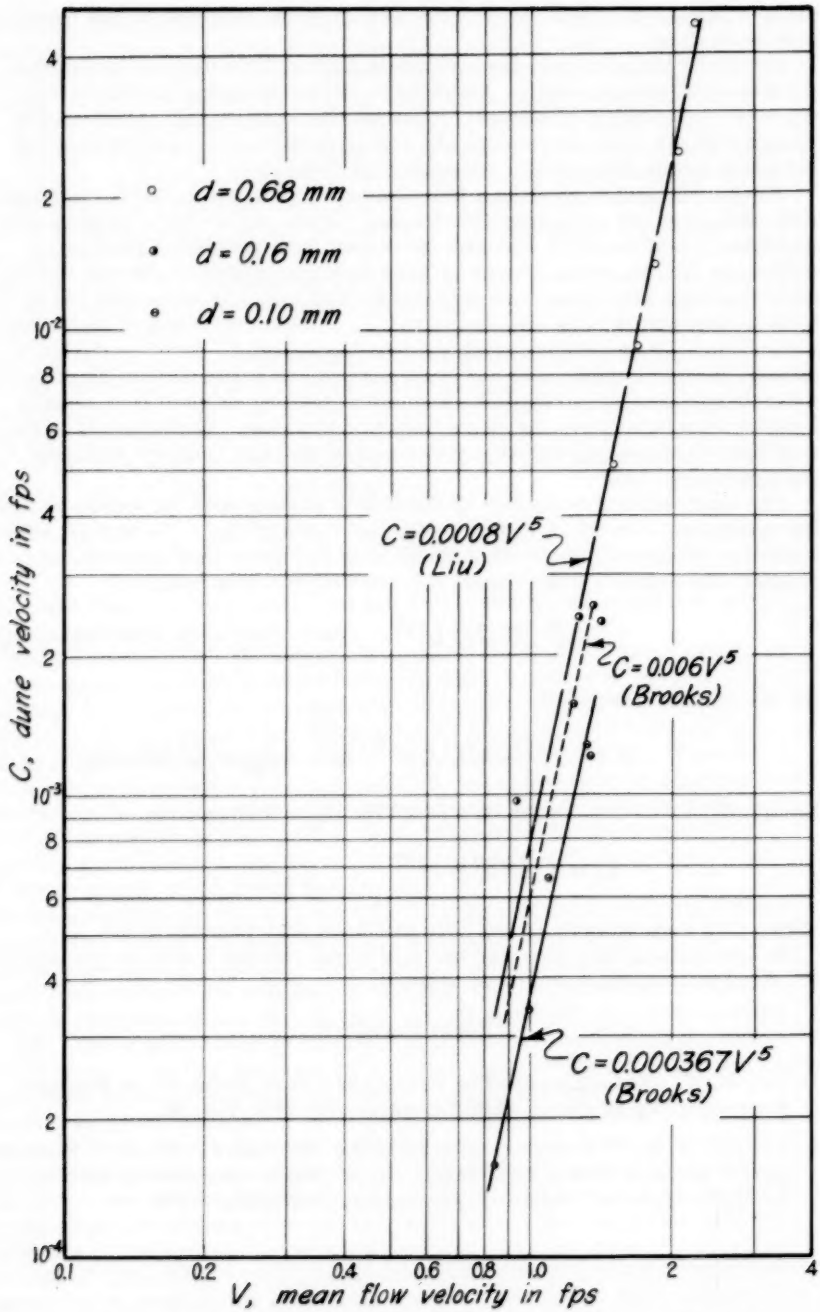

Fig. a. Relation between dune velocity and mean flow velocity.

PIN-NAM LIN[1]—The principal issue raised in the author's paper is whether "there is only one equilibrium rate of flow and sediment transportation rate corresponding to each combination of bed hydraulic radius and slope."(1) This statement should be read with the understanding that, among other factors, given temperature, channel, and sediment are implied, otherwise the statement will be trivial. The author concluded on the basis of Figs. 6 and 7 that more than one equilibrium condition may exist for a given set of depth, slope and others. He specifically cited Run 3 and Run 9 to support his contention. Unfortunately, Table L1 prepared from the author's data

Table L1

Run No.	U ft/sec	$\overline{C}$ gr/l	d ft	S	T C
3	2.04	1.95	0.243	0.0025	22
7	2.04	2.15	0.243	0.0021	31.5
8	1.75	1.5	0.24	0.0023	27.5
9	1.35	1.1	0.245	0.0026	27.5

shows that Runs 3 and 9 were made at different temperatures. The effect of temperature may be clearly seen by comparing Runs 3 and 7. The slope of Run 3, when corrected for the effect of water temperature by linear interpolation, might be as low as 0.0023, so that one would have been tempted to hastily dismiss Mr. Brooks' argument. Inspection of Table L1, however, shows that when corrected for temperature effect, Run 3 could have practically the same value of d and S as Run 8. Figs. 6 and 7 are such that it is difficult to determine whether there are any other runs of the same nature as Runs 3 and 8, except for possibly Run 21 (U=2.10, $\overline{C}$= 4.85) and Run 27 (U=0.99, $\overline{C}$=1.35). It appears to the writer, therefore, that altogether not more than four runs are plainly supporting the author's contention. Of these four runs, Run 8 and Run 27 are for the case of dunes on the bed, and Run 8 involves a "meandering" bed, which is not really in equilibrium.

In view of the large errors normally encountered in the experimental study of sediment transport, it is not safe to draw firm conclusions, even if only qualitative, on the basis of a few runs. A better procedure to arrive at any conclusion would be to search for a method by which the data may be presented to indicate general trends. Such trends, if existing, will then be representative of the entire group of data. Any conclusion drawn on the basis of these general trends of behavior would be more significant than that drawn on the basis of a few runs. Considerations of the effect of density gradient on vertical mixing have led the writer to the use of the following family of dimensionless groups in the analysis of sediment data:

$$\frac{w\overline{C}}{UF^2}\ ,\quad \frac{wC}{UF^2}\ ,\quad \frac{w\overline{C}}{UB}\ ,\quad \frac{wC}{U_* S}\ .$$

The detailed reasoning leading to the formulation of such parameters has been given in a report by Mr. Barton and the writer.(2) When the slopes of the author's data presented in Figs. 6 and 7 are plotted against $\frac{w\overline{C}}{UF^2}$ as shown in Fig. L1, interesting trends of the data can be seen.

1. Dept. of Concrete and Hydraulics, Corbett, Tinghir & Co., New York, N. Y.

According to Fig. L1, as long as $w\bar{C}/UF^2$ is smaller than 0.07, there is a unique relationship between S and $w\bar{C}/UF^2$ for both sizes of sand tested. For higher values of $w\bar{C}/UF^2$ than 0.07, the relationship between S and $w\bar{C}/UF^2$ is far more complicated. For each sand and each type of bed configuration (i.e. either flat bed or a bed covered with dunes), a curve will in time branch out from the main curve.

In terms of symbols, the following functions of the most essential and mutually independent variables may be written:

$$S = f_1 (U, d, \mu, \rho, g, w) \quad \ldots\ldots\ldots \quad (L1)$$

and,

$$\bar{C} = f_2 (U, d, \mu, \rho, g, w) \quad \ldots\ldots\ldots \quad (L2)$$

From (L1) and (L2),

$$S = f_3 (U, d, \rho, \bar{C}, g, w) \quad \ldots\ldots\ldots \quad (L3)$$

Since now the function f_3 is to be determined by Fig. L1, it should be obvious that the uniqueness or multiplicity of the function f_3 is determined by the uniqueness or multiplicity of the relationship between S and wC/UF^2. In the region where $w\bar{C}/UF^2 < 0.07\pm$, f_3 is unique no matter which six of the seven pertinent variables are chosen as independent. Since this is the region in which most of the existing laboratory and field data fall, it is clear why a unique equilibrium condition has always been observed under a given set of sufficient conditions. In the region of branching curves, f_3 is unique only for a given material and a given type of bed configuration, so that in this region f_3 is not unique no matter which six of the pertinent variables are chosen as independent. In other words, for the same size of sand, there will be two forms of f_3, one each for the case of flat bed and the case of a bed covered with dunes, respectively. To be more specific, one may write for this region

$$S = f_4 (w\bar{C}/UF^2, w/U, \text{bed configuration}).$$

This is because a horizontal or vertical line drawn in the region of branching curves may have two intersections with curves for the same size of sediment. Thus in this region of branching curves, the author's conclusion (1)[3] with due modification to take other essential variables into account is supported by the author's own data; whereas his conclusion (2) is not compatible with his data.

The behavior of data in the region of branching curves appears to result from the fact that in this region the effect of the density gradient on the rate of vertical mixing is great enough to markedly reduce the energy of turbulence, so that for each size and each bed configuration, the turbulence in the flow will in time be appreciably different from that of a "fully developed" shear flow. As the flume used by the author is quite narrow, it is understandable that the author has not presented any data of the Kármán constant. When data collected in a wider flume are analyzed, a plot of Kármán constant against $w\bar{C}/UF^2$ would be found useful as a guide to establish the curves on Fig. L1.

Another important point made by the author is that "the transportation rate could not be expressed as a unique function of the bed shear stress, the channel geometry, and properties of sand." A similar finding has been published by Meyer-Peter and Mueller in 1948.(4) They found that an experimental formula relating the rate of sediment transport and the shearing stress for the case of a flat bed always predicted too high a rate of transport when applied to a bed covered with dunes. Although the Meyer-Peter and Mueller study was primarily intended for the so-called bed-load transport, the fundamental mechanism as far as the rate of sediment transport is concerned should be comparable. From a practical point of view, the question of uniqueness here is perhaps not as important as the question of existance of any function for the bed-material load discharge expressed in terms of bed shear stress and others. The Meyer-Peter and Mueller study of bed load transport furnishes definite proof that such functional relationships do exist. The Colorado A and M College data for 0.18mm sand show a definite functional relationship between $\overline{C}$ and $U/\sqrt{\tau_0/\rho}$ in approximately two-dimensional flow carrying heavy suspended load, τ_0 here being the bed shear stress.(5) With the aid of Mr. Brooks' data, it now appears that two relationships between $\overline{C}$ and $U/\sqrt{\tau_0/\rho}$ may be defined. This point will be dealt with in greater detail in a paper being prepared by the writer and Mr. Barton.

According to Fig. L1, the author's data indicate that, for each size of sediment and each type of bed configuration, there is a critical value of $w\overline{C}/UF^2$, below which there is a unique relationship between S and $w\overline{C}/UF^2$, i.e.,

$$S = f_5 \left(w\overline{C}/UF^2 \right).$$

A study made by Mr. Barton and the writer(6) shows that for each type of bed configuration, one may expect

$$U/U_* = f_6 \left(w\overline{C}/UF^2\right).$$

An interesting deduction is then

$$U/U_* = f_7 \text{ (S , bed configuration) .} \qquad \ldots\ldots \text{(L4)}$$

Similarly, for each size when the critical value of wC/UF^2 is exceeded, one may also expect the existence of a functional relationship such as

$$U/U_* = f_8 \text{ (S , bed configuration).} \qquad \ldots\ldots\ldots \text{(L5)}$$

The writer regards Eqs. L4 and L5 as indicating a very interesting property of the resistance coefficient of an alluvial channel, and believes further investigations of these simplifying expressions may prove worthwhile.

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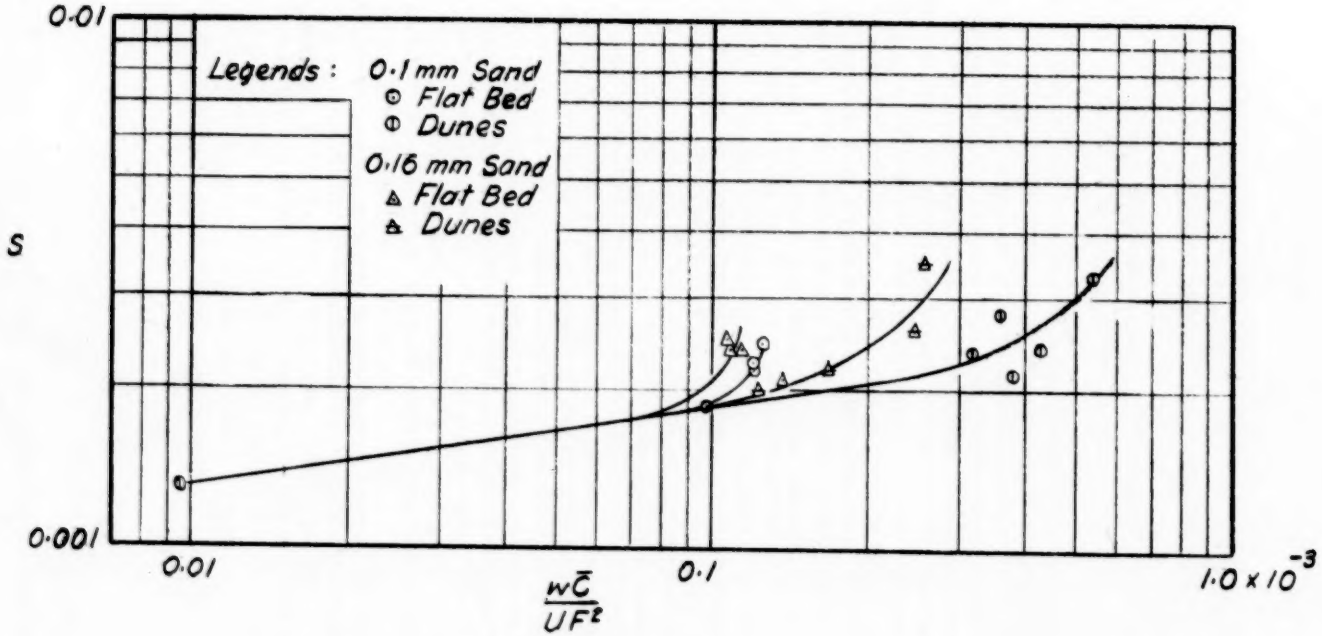

Fig. L1 - A Plot of Brooks' Data

Discussion of
"TIDAL COMPUTATIONS IN SHALLOW WATER"

by J. J. Dronkers and J. C. Schonfeld
(Proc. Paper 714)

G, B. FENWICK,[1] M. ASCE.—The authors have presented and explained with exceptional clarity the intricacies of a variety of computation methods for the mathematical approach to the complicated field of tidal hydraulics.

Comments by the authors also reflect an understanding of the use of hydraulic models for the solution of tidal problems. The paper mentions that a distorted-scale hydraulic model fails to assure true representation of local flow patterns and of the distribution of velocity in a cross section. While this statement is in a very strict sense quite correct, it has nevertheless been well demonstrated in recent tidal models at the Waterways Experiment Station that these two phenomena are reproducible in considerable detail with entirely acceptable accuracy, provided: (1) linear scales are suitably large; (2) scale distortion is not too great for the particular case; and (3) there are available sufficient observed hydraulic and salinity data from the prototype to provide the basis for a fine adjustment of the model roughness.

The major key to the accuracy of a hydraulic tidal model lies in the delicate art of adjusting the model roughness. Adjustment of the surface roughness alone can produce the correct mean velocity and total discharge in any cross section, but not local flow patterns and velocity distributions. However, the judicious use of some surface roughness in proper combination with a few small strips of roughness extending almost through the depth of flow can produce a model capable of reproducing with remarkable accuracy the local flow patterns, the velocity distributions transversely and vertically, and the varying degrees of mixing of salt and fresh water as these phenomena occur in the prototype for all conditions of tide and fresh-water river discharge. If linear scales are appropriate, the roughness strips required will be sufficiently widely spaced to preclude undue interference with measurements of velocities and other quantities in the model.

As the authors point out, a tidal model renders great assistance to the engineer by permitting him to visualize the water motion very clearly and directly, which is particularly important in evaluating the effects of proposed physical changes in an estuary upon its hydraulic regimen. The Netherland's procedure of utilizing both models and the several methods of computation appears to be the ideal approach to the manifold problems encountered in estuarine environments.

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